On the onset of the dark energy era

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The occurrence of the scaling accelerated phase after matter dominance has been shown to be rather problematic for all existing dark energy and modified gravity models. In this paper we consider a cosmic scenario where both the matter particles and scalar field are associated with sub-quantum potentials which make the effective mass associated with the matter particles to vanish at the coincidence time, so that a cosmic system where matter dominance phase followed by accelerating expansion is allowed.

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A recent paper by Amendola, Quartin, Tsujikawa and Waga (hereafter denoted as AQTW) [1] has put all existing models for dark energy in very serious trouble. Actually, if the result obtained by AQTW would be confirmed with full generality, then the whole paradigm of dark energy ought to be abandoned. Such as it happens with other aspects of the current accelerating cosmology, the problem is to some extend reminiscent of the difficulty initially confronted by earliest inflationary accelerating models [2] which could not smoothly connect with the following Friedmann-Robertson-Walker decelerating evolution [3]. As it is well-known, such a difficulty was solved by invoking the new inflationary scenario [4]. In fact, the problem recently posed for dark energy can be formulated by saying that a previous decelerating matter-dominated era cannot be followed by an accelerating universe dominated by dark energy and it is in this sense that it can be somehow regarded as the time-reversed version of the early inflationary exit difficulty. In more technical terms what AQTW have shown is that it is impossible to find a sequence of matter and scaling acceleration for any scaling Lagrangian which can be approximated as a polynomials because a scaling Lagrangian is always singular in the phase space so that either the matter-dominated era is prevented or the region with a viable matter is isolated from that where the scaling acceleration occurs. Ways out from this problem required assuming either a sudden emergence of dark energy domination or a cyclic occurrence of dark energy, both assumptions being quite hard to explain and implement. In this paper we however consider a dark energy model where such problems are no longer present due to some sort of quantum characteristics which can be assigned to particles and radiation in that model.

We start with an action integral that contains all the ingredients of our model. Such an action is a generalization of the one used by AQTW which contains a time-dependent coupling between dark energy and matter and leads to a general Lagrangian that admits scaling solutions formally the same as those derived in Ref. [1]. Setting the Planck mass unity, our Lorentzian action reads

$$S = \int d^4x\sqrt{-g}\left[R + p(X,\phi)\right] + S_m\left[\psi_i, \xi, m_i(V_{SQ}), \phi, g_{\mu\nu}\right] + ST\left(K, \psi_i, \xi\right), \quad (1)$$

where g is the determinant of the four-metric, p is a generically non-canonical general Lagrangian for the dark-energy scalar field ϕ with kinetic term $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, formally the same as the one used in Ref. [1], S_m corresponds to the Lagrangian for the matter fields ψ_i, each with mass m_i, which is going to depend on a sub-quantum potential V_{SQ} in a way that will be made clear in what follows, so as on the time-dependent coupling ξ of the matter field to the dark energy field ϕ. The term ST denotes the surface term which generally depends on the trace on the second fundamental form K, the matter fields ψ_i and the time-dependent coupling $\xi(t)$ between ψ_i and ϕ for the following reasons.

We first of all point out that in the theory being considered the coupling between the matter and the scalar fields can generally be regarded to be equivalent to a coupling between the matter fields and gravity plus a set of potential energy terms for the matter fields. In fact, if we restrict ourselves to this kind of theories, a scalar field ϕ can always be mathematically expressed in terms of the scalar curvature R [5]. More precisely, for the scaling accelerating phase we shall consider a sub-quantum dark energy model (see Refs. [7] and [8]) in which the Lagrangian for the field ϕ vanishes in the classical limit where the sub-quantum potential is made zero; i.e. we take $p = L = -V(\phi)\left(E(x,k) - \sqrt{1-\dot{\phi}^2}\right)$, where $V(\phi)$ is the potential energy and $E(x,k)$ is the elliptic integral of the second kind, with $x = \arcsin\sqrt{1-\dot{\phi}^2}$ and $k = \sqrt{1 - V_{SQ}^2/V(\phi)^2}$, and the overhead dot $\dot{}$ means derivative with respect to time. Using then a potential energy density for ϕ and the sub-quantum medium [note that the sub-quantum potential energy density becomes constant [8] (see later on)], we have for the energy density and pressure, $\rho \propto X(HV_{SQ}/\dot{H})^2 = p(X)/w(t)$, with $H \propto \phi V_{SQ} + H_0$, $\dot{H} \propto \sqrt{2X}V_{SQ}$, where H_0 is constant.

For the resulting field theory to be finite, the condition that $2X = 1$ (i.e. $\phi = C_1 + t$) had to be satisfied [8], and from the Friedmann equation, the scale factor ought to be given by $a(t) \propto \exp\left(C_2 t + C_3 t^2\right)$, with C_1, C_2 and C_3 being constants. It follows then that for at least a flat space-time, we generally have $R \propto 1 + \alpha\phi^2$ (where α is another constant and we have re-scaled time) in that type of theories, and hence the matter fields - scalar field couplings, which can be generally taken to be proportional to $\phi^2\psi_i^2$, turn out to yield $\xi R\psi_i^2 - K_0\psi_i^2$, with K_0 again a given constant. The first term of this expression corresponds to a coupling between matter fields and gravity which requires an extra surface term, and the second one ought to be interpreted as a potential energy term for the matter fields $V_i \equiv V(\psi_i) \propto \psi_i^2$. In this way, for a general theory that satisfied the latter requirement, the action integral (1) should be re-written as

$$\begin{aligned} S = &\int d^4x\sqrt{-g}\left[R(1-\xi\psi_i^2) + p(X,\phi)\right] \\ &+ S_m\left[\psi_i, V_i, m_i(V_{SQ}), g_{\mu\nu}\right] \\ &-2\int d^3x\sqrt{-h}\mathrm{Tr}K(1-\xi\psi_i^2), \end{aligned} \tag{2}$$

in which h is the determinant of the three-metric induced on the boundary surface and it can be noticed that the scalar field ϕ is no longer involved at the matter Lagrangian. We specialize now in the minisuperspace that corresponds to a flat Friedmann-Robertson-Walker metric in conformal time $\eta = \int dt/a(t)$

$$ds^2 = -a(\eta)\left(-d\eta^2 + a(\eta)^2 d\mathrm{x}^2\right), \tag{3}$$

with $a(\eta)$ the scale factor. In this case, if we assume a time-dependence of the coupling such that it reached the value $\xi(\eta_c) = 1/6$ at the coincidence time η_c and choose suitable values for the arbitrary constants entering the above definition of R in terms of ϕ^2, then the action at that coincidence time would reduce to

$$\begin{aligned} S = \frac{1}{2}\int d\eta &\left[a'^2 - \sum_i(\chi_i'^2 - \chi_i^2)\right. \\ &\left. + a^4\left(p(X,\phi) + \sum_i m_i(V_{SQ})^2\right)\right], \end{aligned} \tag{4}$$

where the prime $'$ denotes derivative with respect to conformal time η and $X = \frac{1}{2a^2}(\phi')^2$. Clearly, the fields χ_i would then behave like though if they formed a collection of conformal radiation fields were it not by the presence of the nonzero mass terms m_i^2 also at the coincidence time. If for some physical cause the latter mass terms could all be made to vanish at the coincidence time, then all matter fields would behave like though they were a collection of radiation fields filling the universe at around the coincidence time and there would not be the disruption of the evolution from a matter-dominated era to a stable accelerated scaling solution of the kind pointed out by AQTW, but the system smoothly entered the accelerated regime after a given brief interlude where the matter fields behave like pure radiation. In what follows we shall show that in the sub-quantum scenario considered above such a possibility can actually be implemented.

At the end of the day, any physical system always shows the actual quantum nature of its own. One of the most surprising implications tough by dark energy and phantom energy scenarios is that the universal system is not exception on that at any time or value of the scale factor. Thus, we shall look at the particles making up the matter fields in the universe as satisfying the Klein-Gordon wave equation [6] for a Bohmian quasi-classical wave function [7] $\Psi_i = R_i\exp(iS_i/\hbar)$, where we have restored an explicit Planck constant, R_i is the probability amplitude for the given particle to occupy a certain position within the whole homogeneous and isotropic space-time of the universe, as expressed in terms of relativistic coordinates, and S_i is the corresponding classical action also defined in terms of relativistic coordinates.

Taking the real part of the expression resulting from applying the Klein-Gordon equation to the wave function Ψ_i, and defining the classical energy as $E_i = \partial_i S/\partial t$ and the classical momentum as $p_i = \nabla S_i$, one can then derive the modified Hamilton-Jacobi equation

$$E_i^2 - p_i^2 + V_{SQi}^2 = m_{0i}^2, \tag{5}$$

where V_{SQi} is the relativistic version of the so-called sub-quantum potential [7] which is here given by

$$V_{SQi} = \hbar\sqrt{\frac{\nabla^2 R_i - \ddot{R}_i}{R_i}}, \tag{6}$$

that should also satisfy the continuity equation (i.e. the probability conservation law) for the probability flux, $J = \hbar\ \mathrm{Im}(\Psi^*\nabla\Psi)/(m\mathrm{V})$ (with $\mathrm{V} \propto a^3$ the volume), stemming from the imaginary part of the expression that results by applying the Klein-Gordon equation to the wave equation Ψ. Thus, if the particles are assumed to move locally according to some causal laws [7], then the classical expressions for E_i and p_i will be locally satisfied. Therefore we can now interpret the cosmology resulting from the above formulae as a classical description with an extra sub-quantum potential, and average Eq. (5) with a probability weighting function for which we take $P_i = |R_i|^2$, so that

$$\begin{aligned} &\int\int\int dx^3 P_i\left(E_i^2 - p_i^2 + V_{SQi}^2\right) \\ &= \langle E_i^2\rangle_{\mathrm{av}} - \langle p_i^2\rangle_{\mathrm{av}} + \langle V_{SQi}^2\rangle_{\mathrm{av}} = \langle m_{0i}^2\rangle_{\mathrm{av}}, \end{aligned} \tag{7}$$

with the averaged quantities coinciding with the corresponding classical quantities and the averaged total sub-quantum potential squared being given by $\langle V_{SQ}^2\rangle_{\mathrm{av}} = \hbar^2\left(\langle\nabla^2 P\rangle_{\mathrm{av}} - \langle\ddot{P}\rangle_{\mathrm{av}}\right)$.

It is worth noticing that in the above scenario the velocity of the matter particles should be defined to be

given by

$$\langle v_i\rangle_{\rm av}=\frac{\langle p_i^2\rangle_{\rm av}^{1/2}}{\left(\langle p_i^2\rangle_{\rm av}+\langle m_{0i}^2\rangle_{\rm av}-\langle V_{SQi}^2\rangle_{\rm av}\right)^{1/2}}. \quad (8)$$

It follows that in the presence of a sub-quantum potential, a particle with nonzero rest mass $m_{0i}\neq 0$ can behave like though if was a particle moving at the speed of light (i.e. a radiation massless particle) provided $\langle m_{0i}^2\rangle_{\rm av}=\langle V_{SQi}^2\rangle_{\rm av}$. Thus, if we introduce an effective particle rest mass $m_{0i}^{\rm eff}=\sqrt{\langle m_{0i}^2\rangle_{\rm av}-\langle V_{SQi}^2\rangle_{\rm av}}$, then we get that the speed of light again corresponds to a zero effective rest mass. It has been noticed [8], moreover, that in the cosmological context the averaged sub-quantum potential defined for all existing radiation in the universe should be regarded as the cosmic stuff expressible in terms of a scalar field ϕ that would actually make up our scaling dark-energy solution. At the coincidence time, that idea should actually extend in the present formalism to also encompass in an incoherent way, together with the averaged sub-quantum potential for CMB radiation, the averaged sub-quantum potential for matter particles, as a source of dark energy. On the other hand, it has been pointed out as well [8] that the sub-quantum potential ought to depend on the scale factor $a(t)$ in such a way that it steadily increases with time, being the sub-quantum energy density satisfying the above continuity equation what keeps constant along the whole cosmic evolution.

Assuming the mass m_i appearing in the action (4) to be an effective particle mass, it turns out that the onset of dark energy dominance would then be precisely at the coincidence time when $\langle V_{SQi}^2\rangle_{\rm av}\equiv\langle V_{SQi}(a)^2\rangle_{\rm av}$ reached a value which equals $\langle m_{0i}^2\rangle_{\rm av}$ and all the matter fields behaved in this way like a collection of radiation fields which are actually irrelevant to the issue of the incompatibility of the previous eras with a posterior stable accelerated current regime. In this case, the era of matter dominance can be smoothly followed by the current accelerated expansion where all matter fields would effectively behave like though if they cosmologically were tachyons. This interpretation would ultimately amount to the unification of dark matter and dark energy, as the dark energy model being dealt here with is nothing but a somehow quantized version of tachyon dark energy [9], so that one should expect both effective tachyon matter and tachyon dark energy to finally decay to dark matter, so providing a consistent solution to the cosmic coincidence problem.

Now, from our action integral (4) one can derive the equation of motion for the field ϕ; that is (See also Refs. [10] and [11])

$$\ddot{\phi}\left(p_X+2Xp_{XX}\right)+3Hp_X\dot{\phi}+2Xp_{X\rho}-p_\phi=\frac{\delta S}{a^3\delta\phi}, \quad (9)$$

where we have restored the cosmic time t, using the notation of Refs. [1], [10] and [11], so that a suffix X or ϕ denotes a partial derivative with respect to X or ϕ, respectively, and now the last coupling term is time-dependent. Note that if we confine ourselves to the theory where $a(t)$ accelerates in an exponential fashion and $\dot{\phi}^2=1$ then the first term of this equation would vanish. Anyway, in terms of the energy density ρ for the scalar field ϕ the above general equation becomes formally the same as that which was derived in Ref. [1]

$$\frac{d\rho}{dN}+3(1+w)\rho=-Q\rho_m\frac{d\phi}{dN}, \quad (10)$$

with ρ_m the energy density for the matter field, $N=\ln a$, and $Q=-\frac{1}{a^3\rho_m}\frac{\delta S_m}{\delta\phi}$. We can then derive the condition for the existence of scaling solutions for time-dependent coupling which, as generally the latter two equations are formally identical to those derived by AQTW, is the same as that was obtained by these authors. Hence, we have the generalized master equation for p [1]

$$\left[1+\frac{2dQ(\phi)}{\lambda Q^2 d\phi}\right]\frac{\partial\ln p}{\partial\ln X}-\frac{\partial\ln p}{\lambda Q\partial\phi}=1, \quad (11)$$

whose solution was already obtained by AQTW [1] to be:

$$p(X,\phi)=XQ(\phi)^2g\left(XQ(\phi)^2e^{\lambda\kappa(\phi)}\right) \quad (12)$$

where g is an arbitrary function, λ is a given function of the parameters of the equations of state for matter and ϕ and the energy density for ϕ, being $\kappa=\int^\phi Q(\xi)d\xi$ (see Ref. [1]). In the phase space we then have an equation-of-state effective parameter for the system $w_{\rm eff}=-1-\frac{2\dot{H}}{3H^2}=gx^2+z^2/3$, with H the Hubble parameter and x and z respectively being $x=\dot{\phi}/(\sqrt{6}H)$ and $z=\sqrt{\rho_{\rm rad}/(3H^2)}$. At the coincidence time where we have just radiation ($z\neq 0$ and $\rho_m=\rho_{\rm rad}$) the effective equation of state is [1] $w_{\rm eff}=1/3$. Hence at the coincidence time interval we can only have radiation, neither matter or accelerated expansion domination, just the unique condition that would allow the subsequent onset of the accelerated expansion era where conformal invariance of the field χ no longer holds.

Thus, it appears that in the considered model a previous matter-dominated phase can be evolved first into a radiation phase at a physical regular coincidence short stage which is then destroyed to be finally followed by the required new, independent phase of current accelerating expansion. This conclusion can be more directly drawn if one notices that there is no way by which the general form of the Lagrangian (12) can accommodate the Lagrangian final form $L\equiv p=f(a,\dot{a})\dot{\phi}^2V_{SQ}^2$ which characterizes sub-quantum dark energy models whose pressure p vanishes in the limit $V_{SQ}\to 0$. It thus appears that at least these models can be taken to be counter examples to the general conclusion that current dark-energy and modified gravity models (see however Ref. [12]) are incompatible with the existence of a previous matter-dominated phase, as suggested in Ref. [1].

We finally notice, moreover, that the kind of subquantum dark energy theory providing the above counter example is one which shows no classical analog (i.e. the Lagrangian, energy density and pressure are all zero in the classical limit $\hbar \to 0$) and is thereby most economical of all. Thus, the above conclusion can also be stated by saying that, classically, a previous phase of matter dominance is always compatible with the ulterior emergence of a dominating phase made up of "nothing". In this way, similarly to as the abrupt, unphysical exit of the old inflationary problem was circumvented by introducing [4] a scalar field potential with a flat plateau leading to a "slow-rollover" phase transition, the abrupt disruption of the scaling phase after matter dominance can be also avoided by simply considering a vanishing scalar field potential that smooths the transition and ultimately makes it to work.

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